On the one-endedness of graphs of groups

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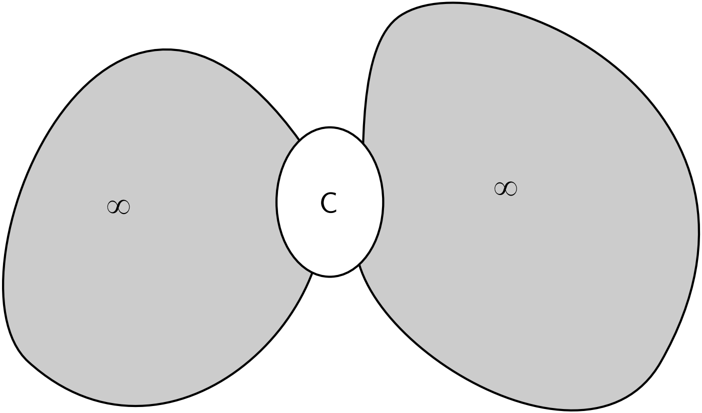
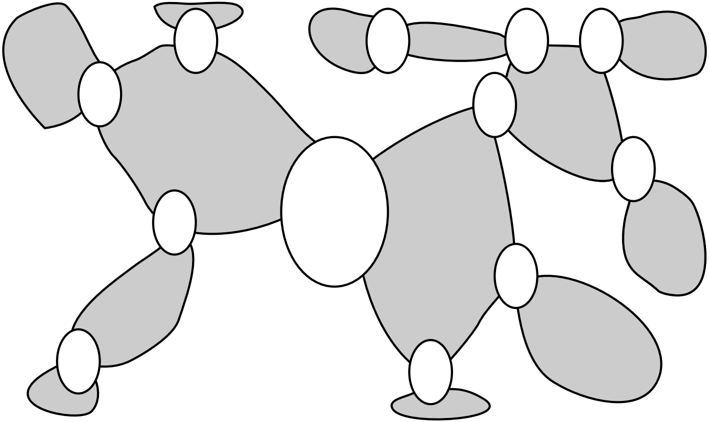
Ends

Let $G = \langle S \rangle$ be generated by the finite set S. We say G is *many ended* if its Cayley graph $\mathrm{Cay}(G, S)$ can be separated into at least two infinite connected components by the removal of a finite set C.



Stallings's Theorem



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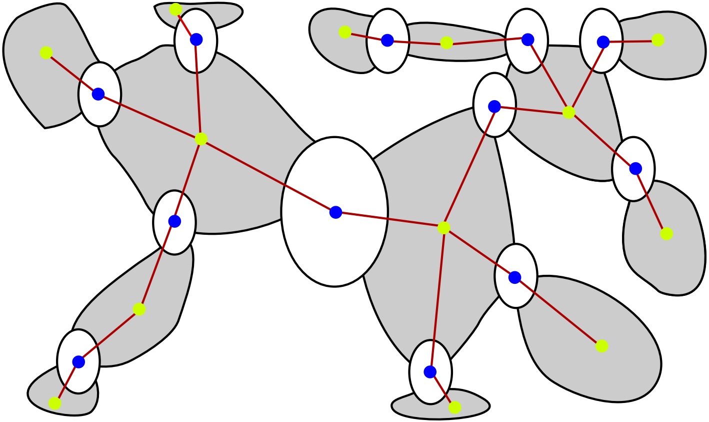

Stallings's theorem tells us that the finite cut set C can be chosen to give a G-tree. In particular this tree has finite edge stabilizers.

Bass-Serre theory

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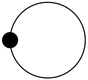
   $G \backslash T =$ 

   then G is an HNN extension, i.e. $G = \langle A, t \mid t^{-1}at = \phi(a); a \in C \rangle$ where $C, C' \leqslant A$ and $\phi : C \xrightarrow{\sim} C'$. We also write $G = A*_C^t$.

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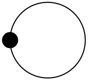
   $G \backslash T =$ 

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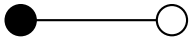
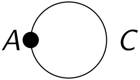
   $G \backslash T =$ 

   then G is an amalgamated free product, i.e. $G = A *_C B$ where $A \geqslant C \leqslant B$.

Terminology

In the previous examples the groups A, B are called *vertex groups* and the groups C are called *edge groups*. We will sometimes label the vertices and edges of $G\backslash T$ by the corresponding groups.



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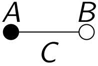

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A working definition

We say G *is one ended* if it does not split as an amalgamated free product or an HNN extension with finite edge group.

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Let $H \leqslant G$. We say G is *one ended relative to* H if it is impossible to decompose G non-trivially as a graph of groups (and in particular an a.f.p. or HNN extension) with finite edge groups such that H is conjugate into one of the vertex groups.

Relative one-endedness, an example

Let $H = \langle [a, b] \rangle \leqslant \mathbb{F}(a, b)$, where $[a, b] = a^{-1}b^{-1}ab$. We will show that $\mathbb{F}(a, b)$ is one ended relative to H.

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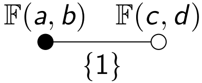
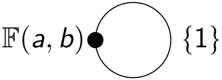
First note that, free groups only have trivial finite subgroups, and for all free products with amalgamation (=free product) and HNN extensions we have



$\approx \mathbb{F}(a, b, c, d)$



$\approx \langle a, b, t \mid \cancel{t^{-1}1t = 1} \rangle = \mathbb{F}(a, b, t)$

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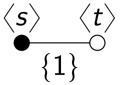
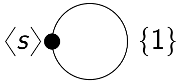
It follows that the only non-trivial splittings of $\mathbb{F}(a, b)$ as an a.f.p. or an HNN extension with finite edge group are



$$= \langle s \rangle * \langle t \rangle = \mathbb{F}(s, t)$$



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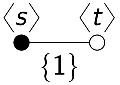

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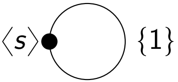

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So if $\mathbb{F}(a,b)$ is not one-ended relative to H we have w.l.o.g. that $[a,b] = s^n \in \langle s \rangle$ for some $n \neq 0$.

Relative one-endedness, an example

Since $s \in \mathbb{F}(a, b)$ is a basis element it is mapped to a basis element via the abelianization $\mathrm{ab} : \mathbb{F}(a, b) \twoheadrightarrow \mathbb{F}(a, b)/[\mathbb{F}(a, b), \mathbb{F}(a, b)] \approx \mathbb{Z}^2$. It therefore follows that $\mathrm{ab}(s) \neq \vec{0}$.

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It follows that H cannot be conjugated into a vertex group of a non-trivial splitting of $\mathbb{F}(a, b)$ with finite edge groups, i.e. $\mathbb{F}(a, b)$ is one-ended relative to H.

A recipe to make one-ended groups

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Theorem (T, Main corollary)

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Answers a question asked privately by John MacKay and Alessandro Sisto. A more general result about the one-endedness of arbitrary graphs of groups will be given later (it is more technical.)

The theorem in action

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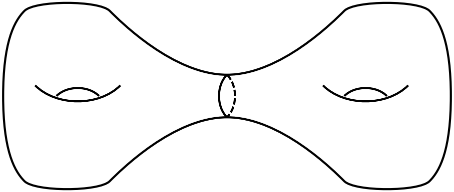

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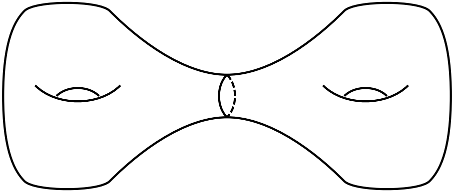
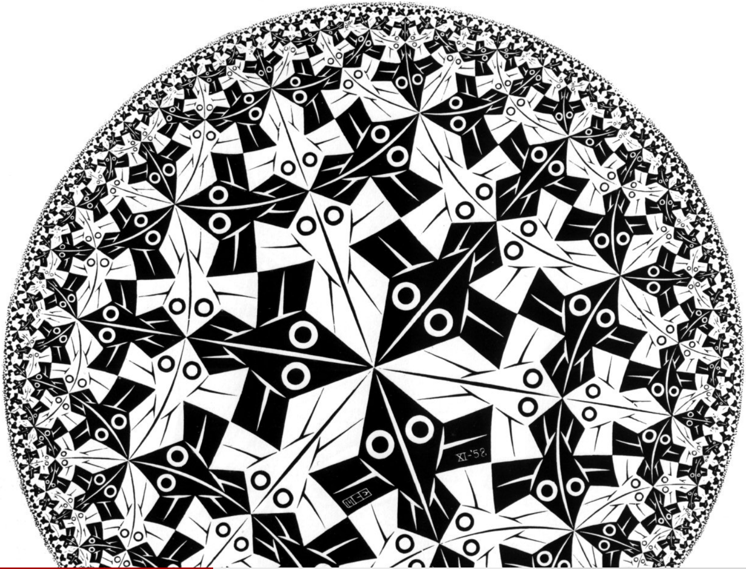


$$\pi_1(\Sigma) = \mathbb{F}(a,b) *_{[a,b]} \mathbb{F}(a,b)$$

and $\pi_1(\Sigma)$ is one ended ...

because $\mathrm{Cay}(\pi_1(\Sigma))$ looks like $\widetilde{\Sigma}$, which looks like ...



Known Corollaries

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Chris Cashen recently generalized this result to virtually free groups with a virtually cyclic splitting. Swarup, and Diao-Feighn, obtained a more technical statement describing *all* splittings of free groups as HNN extensions and amalgamated free products. Using our new machinery we can generalize this to splittings of virtually free groups.

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Theorem (Baumslag)

Let $\langle h_1, \ldots, h_n \rangle = H \leqslant \mathbb{F}$. If there is some word $w(h_1, \ldots, h_n)$ which is not a proper power and not primitive but such that $g = w(h_1, \ldots, h_n) \in \mathbb{F}$ is a proper power, then the set $\{h_1, \ldots, h_n\}$ is not a basis for $H \leqslant \mathbb{F}$; i.e. $\mathrm{rk}(H) < n$.

Novelties

The main Corollary was folklorically known to be true for torsion-free groups and is proved using graph of spaces methods. We use a more abstract structure, namely Guirardel's cocompact core, which can deal with torsion.

Swarup used advanced homological methods which only work without torsion. The proof we will give is elementary, modulo Bass-Serre theory and Guirardel's compact core theorem.

Strategy of proof of the Main Corollary

We have a group G that splits as $G = G_1 *_C G_2$ with C virtually cyclic; this splitting corresponds to an action on a tree T_∞. Suppose that G also splits as $G = A *_F B$ with F finite; and corresponding tree $T_{\mathcal{F}}$.

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By analyzing the actions of G on T_∞ and $T_\mathcal{F}$ simultaneously we will show that, say, G_1 splits non-trivially as a graph of groups with *finite* edge groups in which $C \leqslant G_1$ is conjugate into a vertex group.

Group actions

Let X be some G-complex (e.g. a graph, a square complex). If $S \subset X$ then we write

$$G_s = \{g \in G \mid gS = S\}$$

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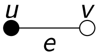
An action $G \curvearrowright X$ is called *without inversion* if whenever $\sigma \subset \rho$ are cells then we have the reverse inclusion of stabilizers $G_\sigma \geqslant G_\rho$. E.g. if in a tree we have:



$$G_u \geqslant G_e \leqslant G_v$$

All our action will be without inversions.

Cocompact and minimal actions, regular subsets

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A subset $S \subset X$ is called *G-regular* if for every $x, y \in S$ there is $g \in G$ such that $gx = y$ if and only if there is some $h \in G_S$ such that $hx = y$.

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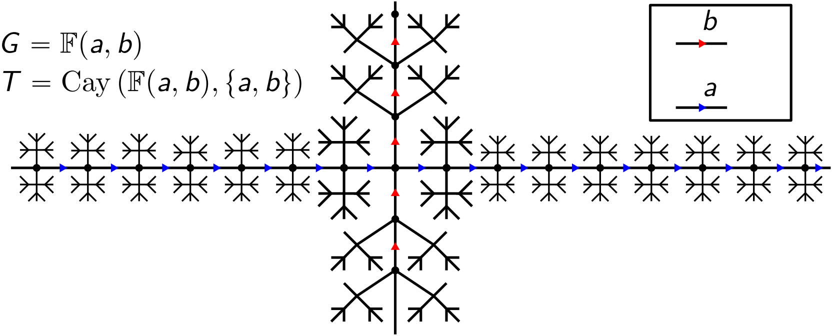
$$G_S\backslash S \hookrightarrow G\backslash X.$$

Examples (cocompact, minimal)

$G = \mathbb{F}(a, b)$

$T = \mathrm{Cay}\left(\mathbb{F}(a, b), \{a, b\}\right)$



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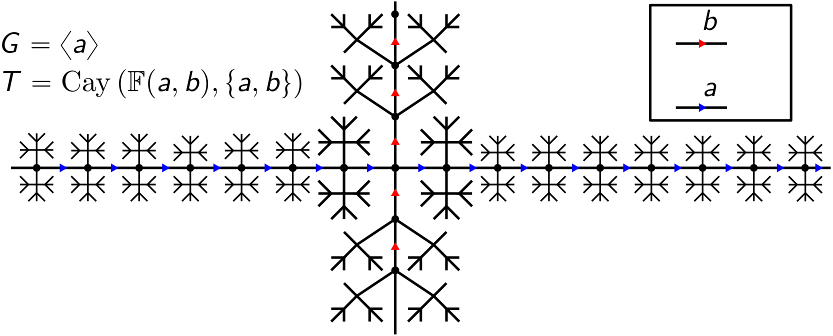

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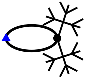
$G \backslash T =$ 

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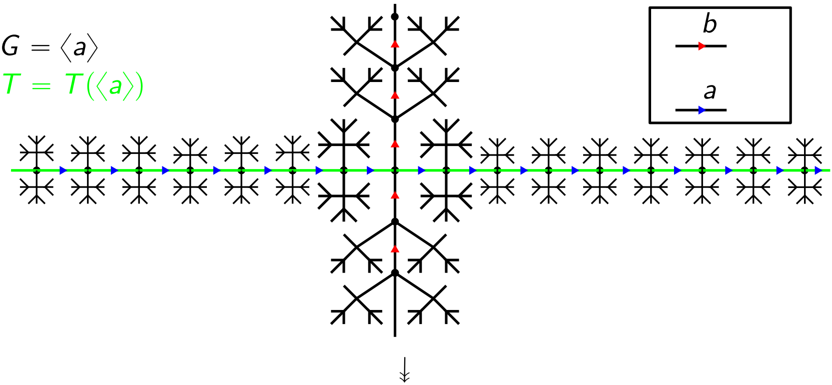
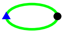


$$G \backslash T(\langle a \rangle) =$$



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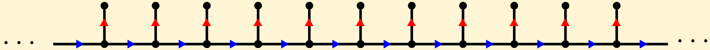

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$\langle a \rangle \backslash S =$  $\nrightarrow$ 

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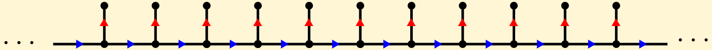

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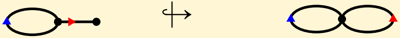

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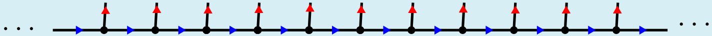

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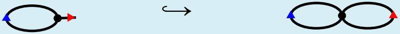

Direct products of trees

To study $G \curvearrowright T_\infty$ and $G \curvearrowright T_{\mathcal{F}}$ simultaneously we can look at the action $G \curvearrowright T_\infty \times T_{\mathcal{F}}$. This product of trees is what is known as a *square complex*.

Direct products of trees

Given a product $T_\infty \times T_\mathcal{F}$ we have natural G-equivariant projections onto the factors

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which restrict naturally to G-invariant subsets $S \subset T_\infty \times T_\mathcal{F}$. The fibers of, say, p_∞ are copies of $T_\mathcal{F}$ (and vice-versa).

The cocompact core

Theorem (Guirardel's Core Theorem)

Let $G \curvearrowright T_1$, $G \curvearrowright T_2$ be two minimal actions of a finitely generated group G on simplicial trees T_1, T_2 with finitely generated edge stabilizers. Then there is a G-invariant subset $\mathcal{C} \subset T_1 \times T_2$ called the core of the action $G \curvearrowright T_1 \times T_2$ which satisfies the following properties:

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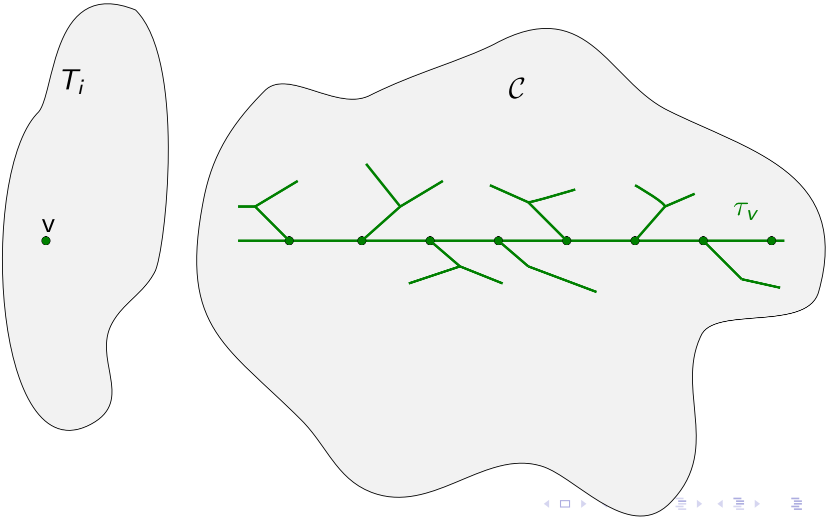
1. *The fibres of the projections $p_i|\mathcal{C} : \mathcal{C} \twoheadrightarrow T_i$ are connected.*
2. *$G \backslash \mathcal{C}$ is compact*

Induced trees



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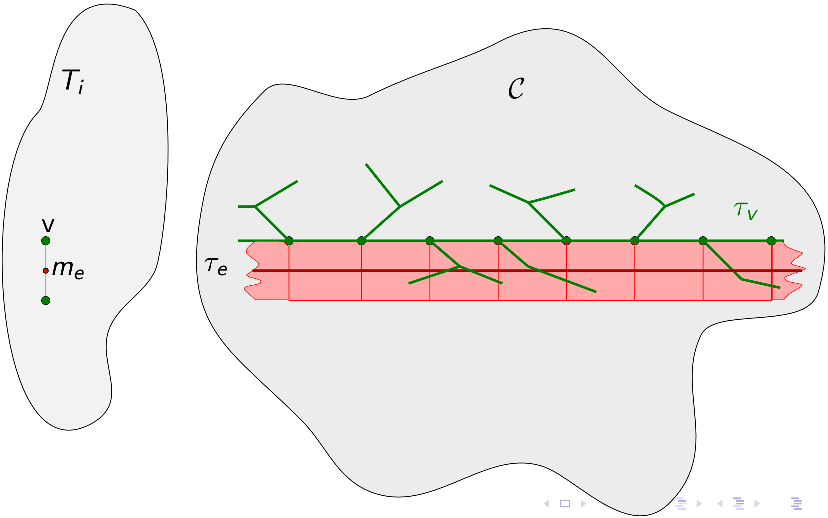
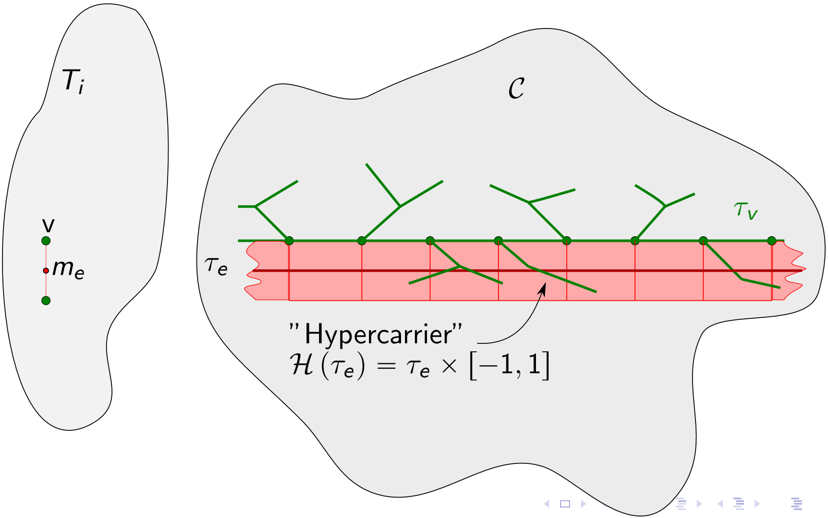


Induced trees



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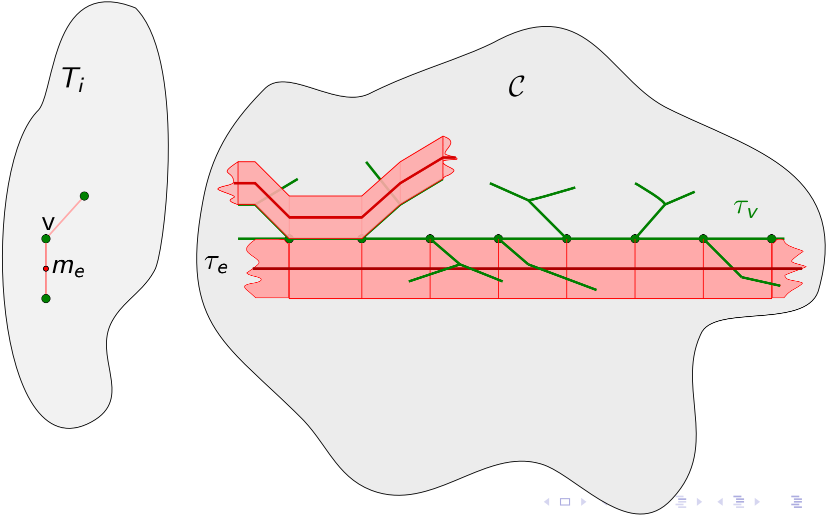

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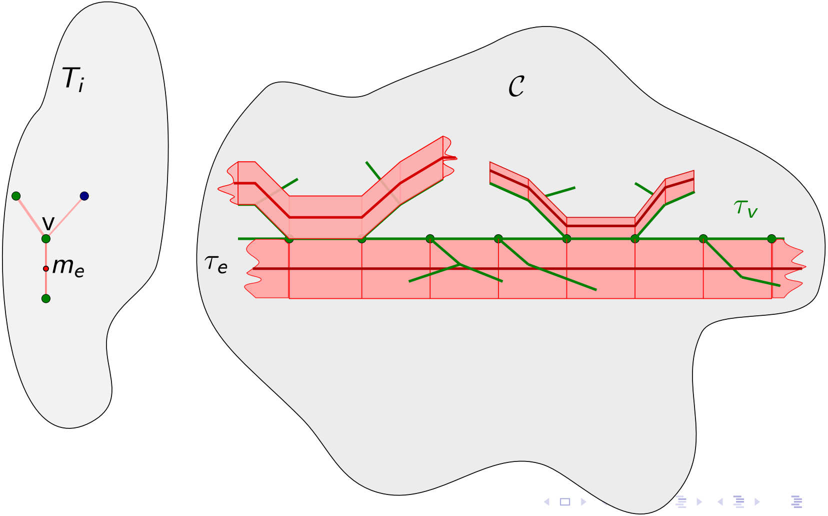

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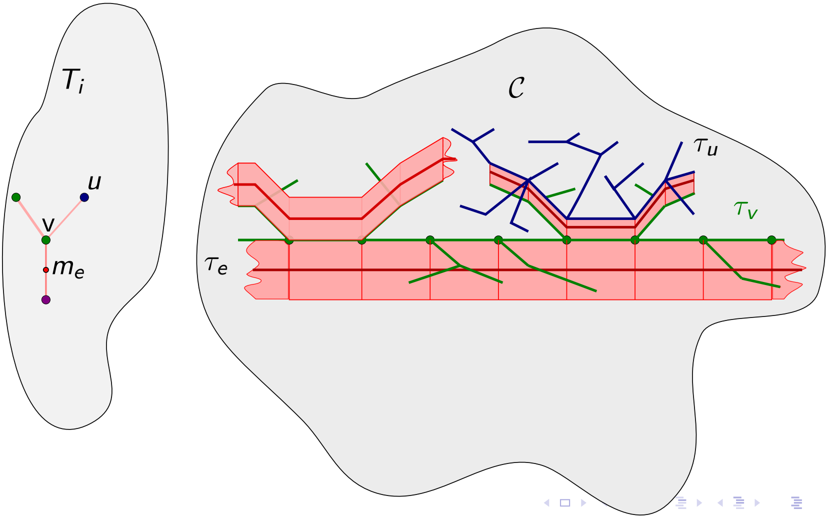

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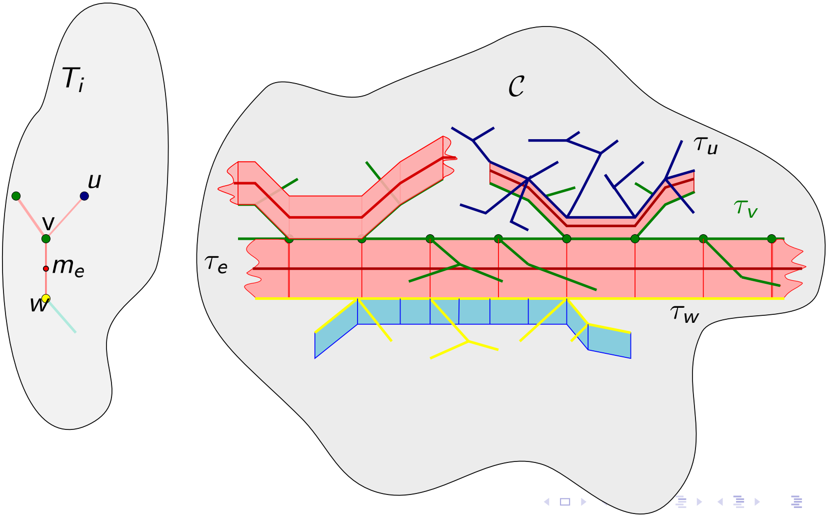

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$$G_v \backslash \tau_v \hookrightarrow G \backslash \mathcal{C} \hookleftarrow G_e \backslash \tau_e$$

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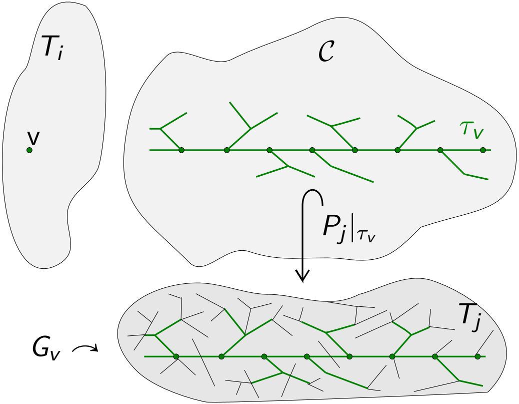
$$\begin{array}{ccccc} G_v\backslash\tau_v & \hookrightarrow & G\backslash\mathcal{C} & \hookleftarrow & G_e\backslash\tau_e \\ & \nwarrow & \uparrow & \nearrow & \\ & & \text{compact} & & \end{array}$$

Therefore τ_v, τ_e are cocompact G_v, G_e-trees.

Induced trees

For $v \in \mathrm{Vertices}(T_i)$, the group G_v may act non-trivially on T_j (the other tree)



The G_v-tree τ_v projects injectively to a G_v-invariant subtree of T_j

What were we doing again?

The decomposition $G = G_1 *_C G_2$ with C virtually $\mathbb{Z}$ is dual to an action $G \curvearrowright T_\infty$ and the decomposition $G = A *_F B$ with F finite is dual to an action $G \curvearrowright T_{\mathcal{F}}$.

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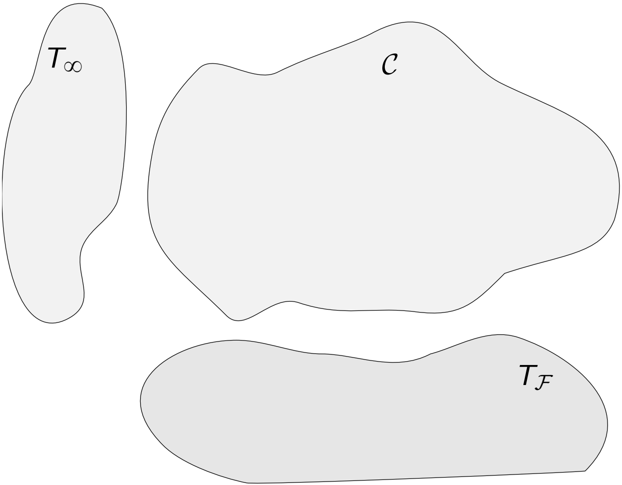
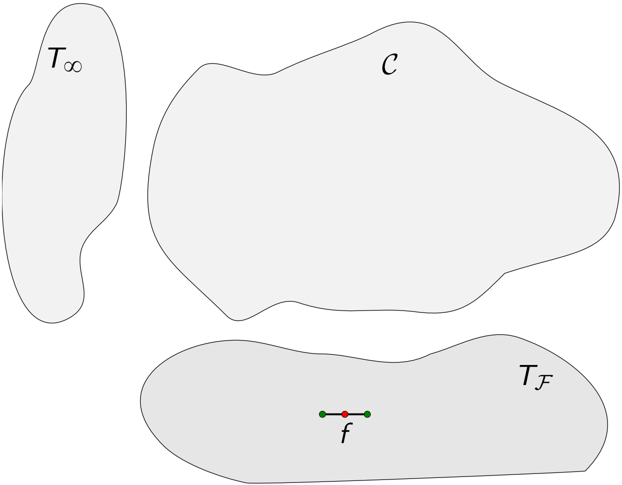
We will assume that the edge group C acts non-trivially on $T_{\mathcal{F}}$, otherwise one of the trees $\tau_v, v \in \mathrm{Vertices}(T_\infty)$ automatically gives the desired splitting.

T_∞ is vertical $T_{\mathcal{F}}$ is horizontal



Vertical hypercarriers are finite



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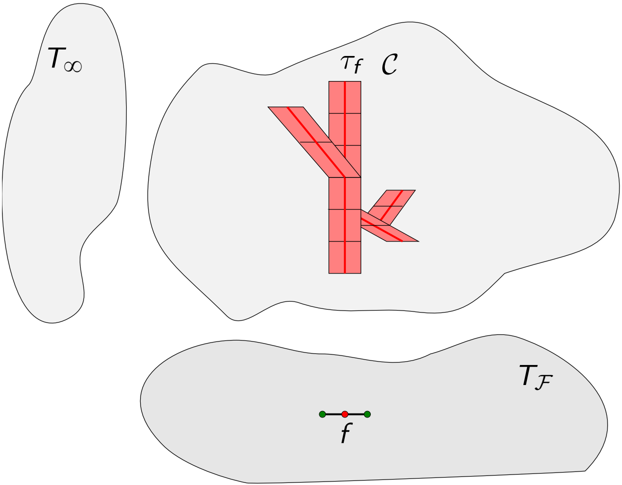

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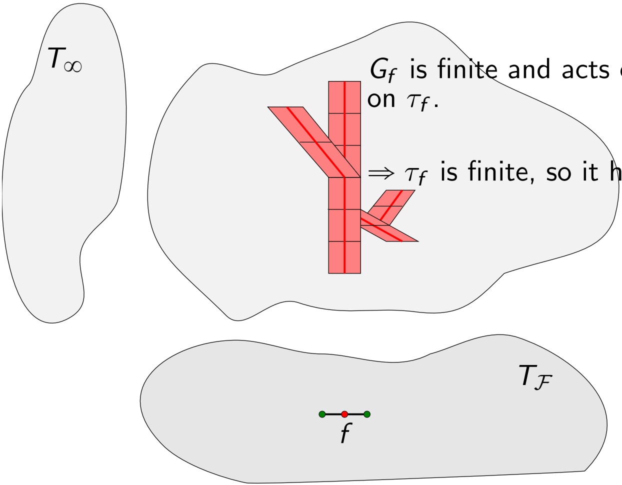
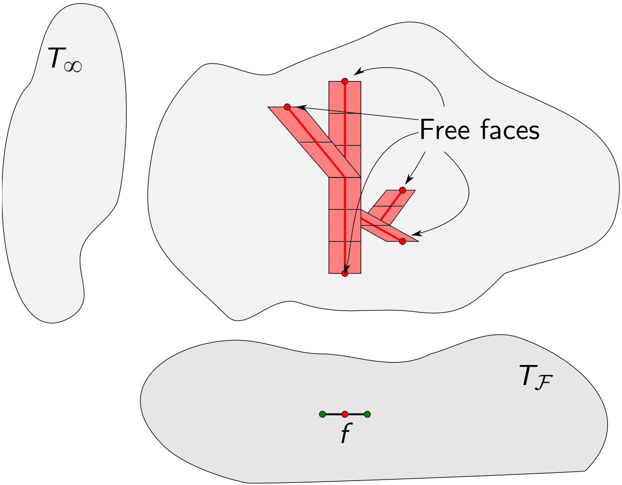


G_f is finite and acts cocompactly on τ_f.

$\Rightarrow$ τ_f is finite, so it has "leaves"

Vertical hypercarriers are finite



Shaving to minimal trees

If necessary, we may G-equivariantly remove squares from the core $\mathcal{C}$ to obtain a connected G-complex $\mathcal{C}_s \subset \mathcal{C}$ called the ∞-minimal core. Which has the property that for every $v \in \mathrm{Vertices}(T_\infty), e \in \mathrm{Edges}(T_\infty)$, the fibres τ_v, τ_e are *minimal* G_v, G_e-trees, respectively.

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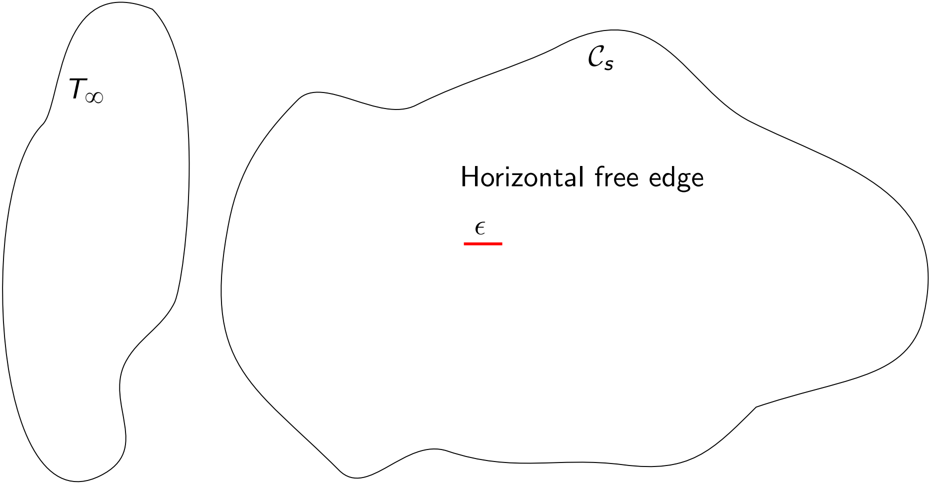
For $f \in \mathrm{Edges}(T_{\mathcal{F}})$, the fibres τ_f may not be connected anymore, but they remain finite forests. The complex $\mathcal{C}_s$ therefore still has horizontal free faces.

Getting a splitting of a T_∞ vertex group



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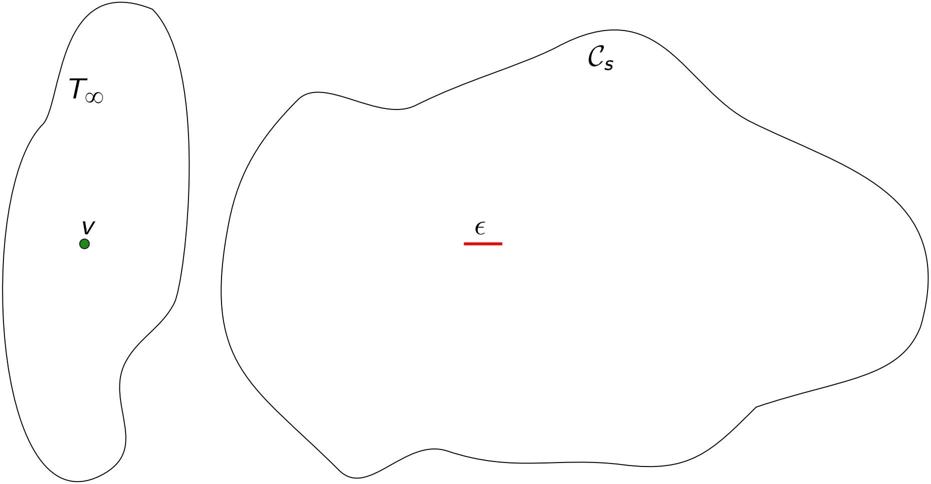

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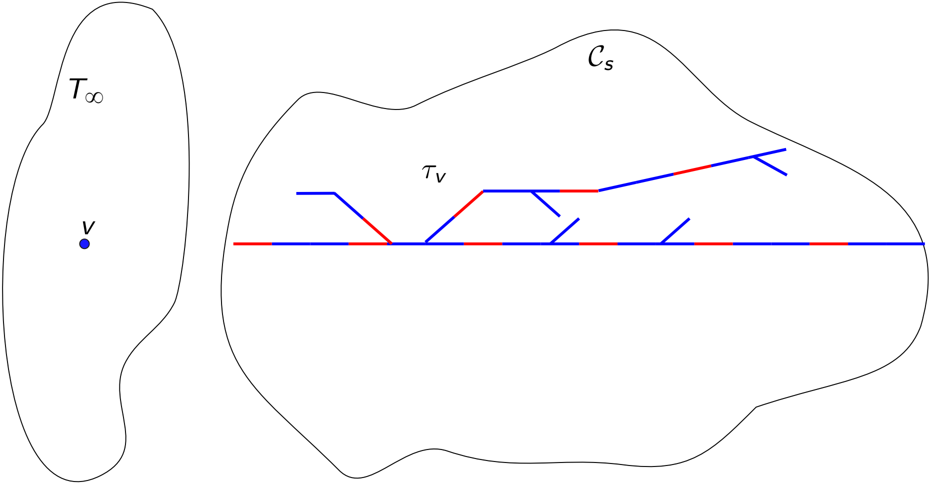

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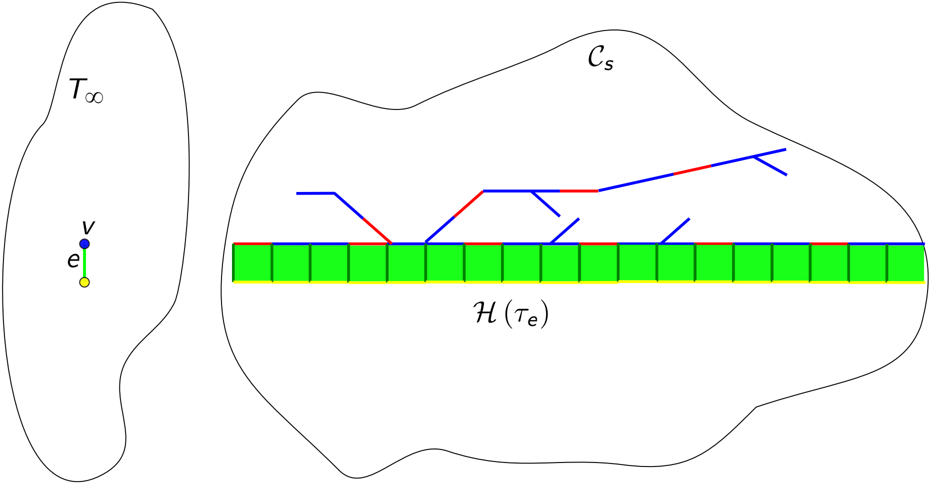

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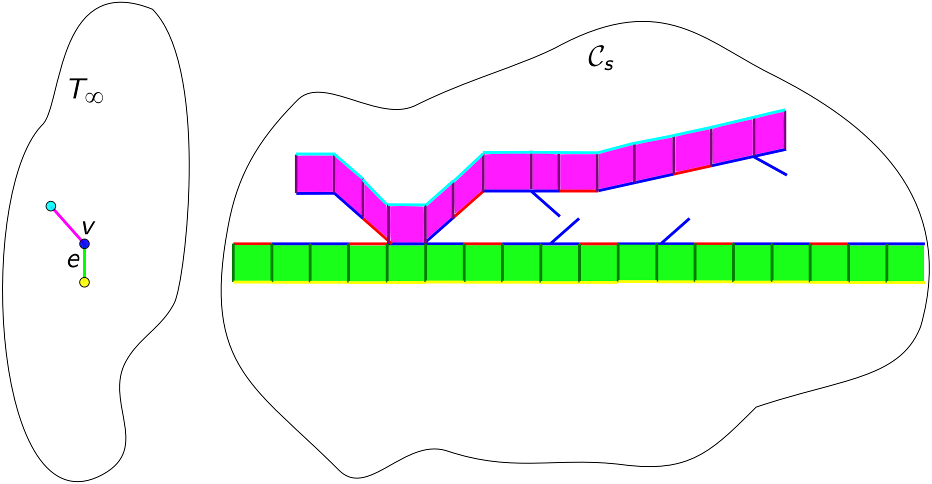

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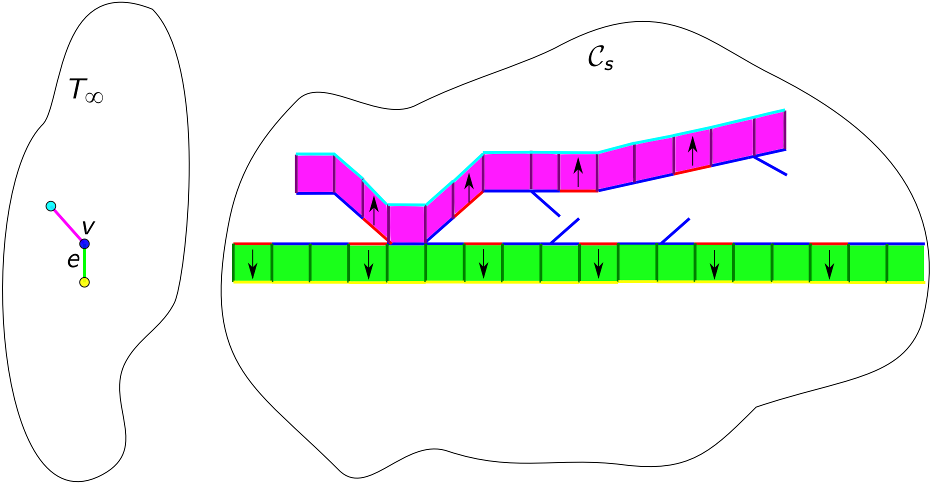

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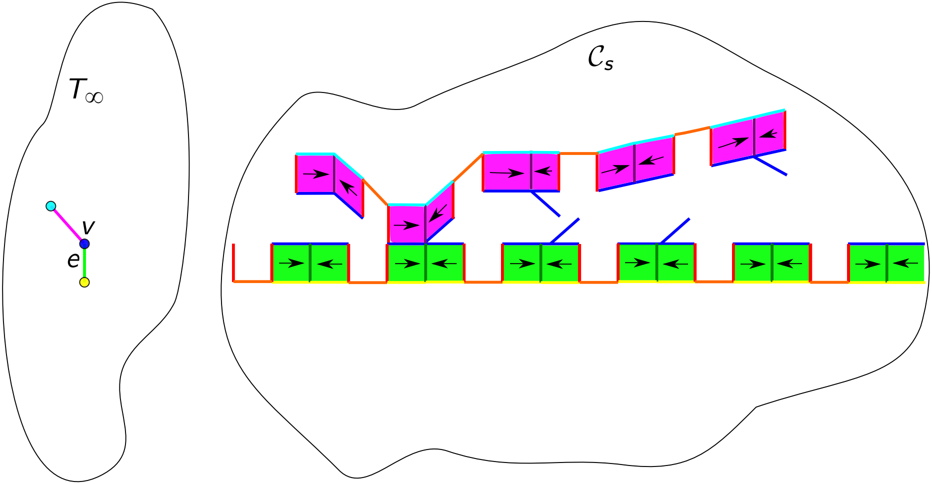

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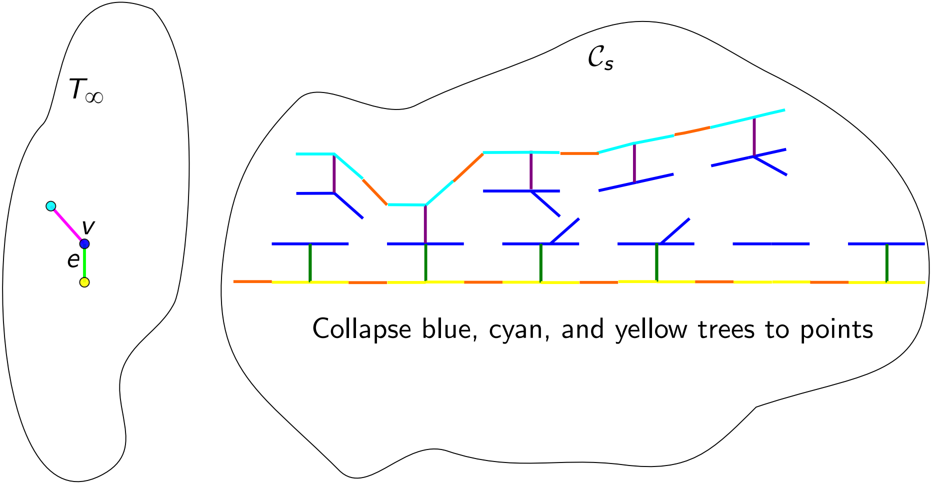

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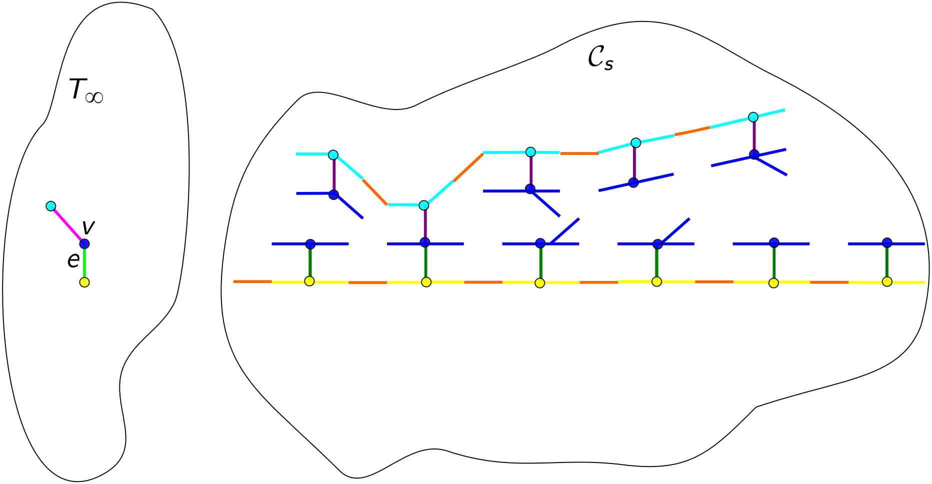

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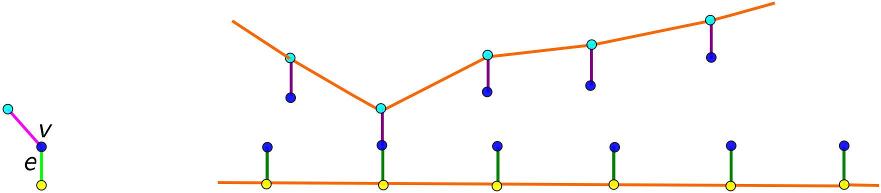

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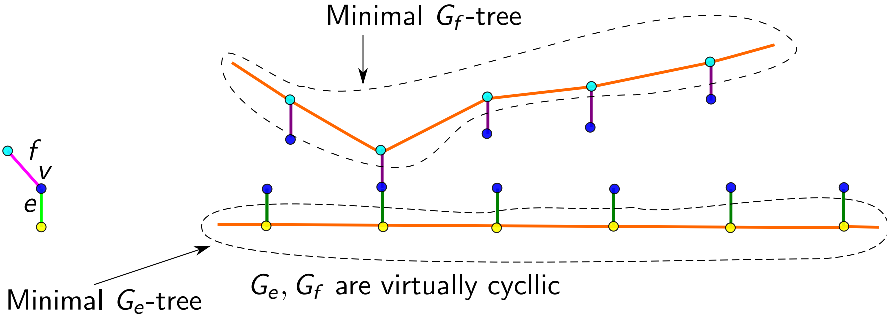
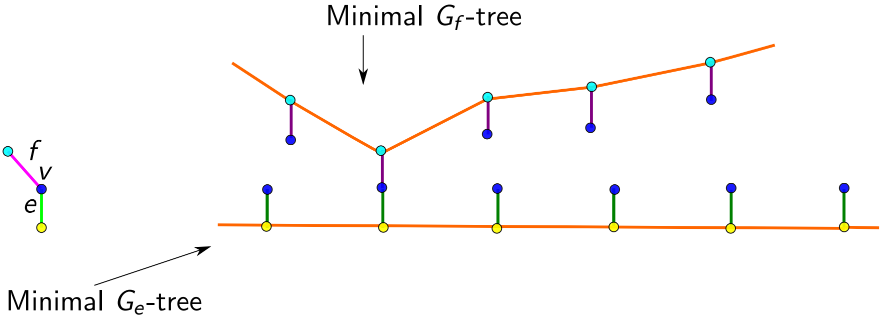


G_e, G_f are virtually cycllic

$\Rightarrow$ cyan and yellow vertex stabilizers are finite

thus green and purple edge have finite stabilizers

Getting a splitting of a T_∞ vertex group



collapse orange segments to points

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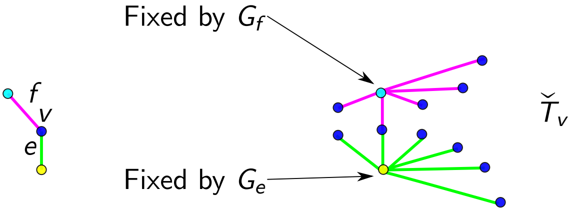

Stabilizers of non-blue vertices of $\check{T}_v$ are conjugate in G to C and edge sabilizers are finite; thus $G_v \sim G_1$ or G_2 is many ended rel. C. This completes the proof.

The general result

It turns out that the procedure that was just described will work for any many ended graph of groups, even if we are working relative to a collection of subgroups.

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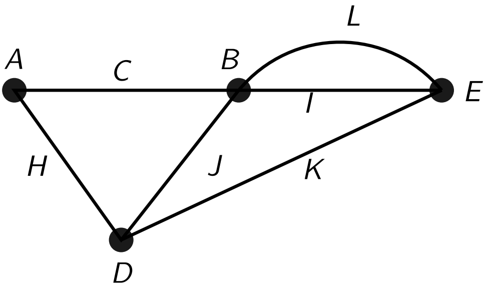
We write $A \prec B$ if A is the vertex group of a splitting of B with finite edge groups. If B is torsion free then $A \prec B \Leftrightarrow A$ is a free factor of B.

The general result

If G is a many ended graph of groups...



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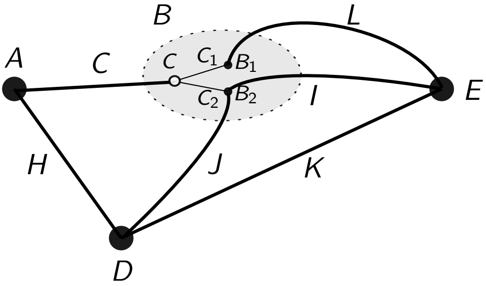

Where $C_1, C_2 \prec C$ and $B_1, B_2 \prec B$.

The general result

By Dunwoody/Linnell accessibility (which holds for large classes of groups) we cannot have infinite chains

$$C \succ C_1 \succ C_2 \succ \dots$$

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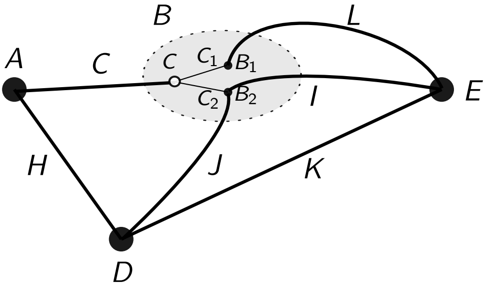

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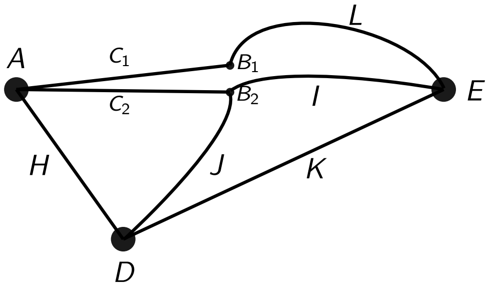

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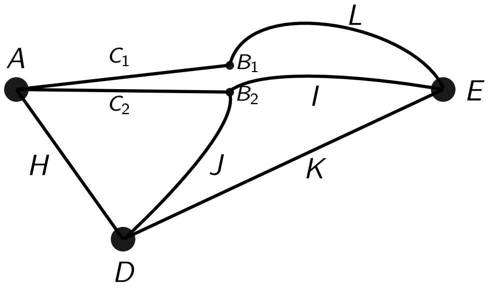

Repeatedly applying this operation will give a Dunwoody decomposition.

Why?

1. It is a theorem of Bonk and Kleiner that the Cayley graph of a one ended δ-hyperbolic group admits a quasi-isometrically embedded $\mathbb{H}^2$. MacKay and Sisto generalized this to relatively hyperbolic groups. There is interesting geometry at work.

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3. This is fundamental progress in dealing with torsion.

Torsion has been problematic (for me.)

Theorem (T)

There is a procedure which takes as input a group presentation $G = \langle X \mid R \rangle$ that is a finite and a solution to the word problem w.r.t. this presentation and outputs whether or not G splits non-trivially as a free product.

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Furthermore, my Strong Accessibility result doesn't work in the presence of $\mathbb{Z}_2 * \mathbb{Z}_2$-type edge groups.

Thank you!